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# **MULTIMEDIA UNIVERSITY**

# FINAL EXAMINATION

TRIMESTER 2, 2015/2016

# **BMS1824 - MANAGERIAL STATISTICS**

(All sections / Groups)

4 MARCH 2016 3.00 p.m. - 5.00 p.m. (2 Hours)

# INSTRUCTIONS TO STUDENTS

- This question paper consists of TWELVE (12) printed pages with: Section A: Ten (10) multiple choice questions (20%)
   Section B: Three (3) structured questions (80%)
- 2. Answer ALL questions.
- 3. Answer Section A in the multiple-choice answer sheet provided and Section B in the answer booklet provided.
- 4. Students are allowed to use non-programmable scientific calculators with no restrictions.

# SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple

choi	ce answer sheet.
1.	The number of vehicles that pass through the New Klang Valley Expressway (NKVE) per day is an example of
	<ul><li>A. Continuous data</li><li>B. Discrete data</li><li>C. Categorical data</li><li>D. Secondary data</li></ul>
2.	Study participants were asked to indicate their ethnic background from a given list such as Malay, Chinese or Indian. This type of variable is classified as
	A. Continuous B. Ordinal C. Nominal D. Discrete
3.	Which of the following statements best describe the relationship between a parameter and a statistic?
	<ul> <li>A. A parameter has a sampling distribution with the statistic as its mean.</li> <li>B. A parameter has a sampling distribution that can be used to determine what values the statistic is likely to have in repeated samples.</li> <li>C. A parameter is used to estimate a statistic.</li> <li>D. A statistic is used to estimate a parameter.</li> </ul>
4.	Which of the following is used to represent a known value for the population variance?
	A. s B. $\sigma$ C. $\sigma^2$ D. $\mu$
	Continued

5. Given that the standard deviation is equal to 0.568, the median equals to 5, and the mean value is 3.5. What is the value of the coefficient of variation?

A. 0.1136

B. 0.162

C. 16.23

D. 0.7

Use the following statement to answer question 6, 7 and 8.

An experiment was conducted to study the choices made in mutual fund selection. Undergraduate and MBA students were presented with different S&P 500 Index funds that were identical except for fees. Suppose 100 undergraduate students and 100 MBA students were selected. Partial results are shown in the following table:

	STUDENT (	GROUP
FUND	Undergraduate	MBA
Highest-cost fund	27	18
Not highest-cost fund	73	82

If a student is selected at random, what is the probability that he or she,

6. Selected the highest-cost fund?

A. 0.450

B. 0.270

C. 0.225

D. 0.135

7. Selected the highest-cost fund and is an undergraduate?

A. 0.225

B. 0.090

C. 0.135

D. 0.635

8. Selected the highest-cost fund or is an undergraduate?

A. 0.815

B. 0.750

C. 0.860

D. 0.725

Continued...

- 9. Which of the following factors is NOT necessary in determining sample size?
  - A. Estimated standard deviation of the population.
  - B. Magnitude of acceptable error.
  - C. Confidence level.
  - D. All of the above are necessary.
- 10. A study has reported on the outcome of the analysis on its hypothesis testing. The report states that the results are statistically significant. This result indicated that
  - A. The null hypothesis is true
  - B. The alternative hypothesis is true
  - C. The p-value is larger than the level of significance
  - D. The test statistic falls outside the rejection region

Continued...

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## SECTION B: STRUCTURED QUESTIONS (80 Marks)

There are THREE questions in this section. Candidates MUST answer ALL THREE questions.

## Question 1 (25 Marks)

a) Malaysia is fast becoming the theme park capital of Southest Asia, with more than 10 theme parks and water parks spread around the country. The data below contains the starting admission price (RM) for one day tickets to 10 theme parks in Malaysia:

58	63	41	42	29
50	(2)	40	40	40

i. Compute the sample mean and median for the above data. Interpret the results.

[8 marks]

ii. Compute the range and sample standard deviation for the data.

[6 marks]

- iii. Using the answers in (i) and (ii), find the coefficient of skewness, S<sub>k.</sub> Interpret the result. [4 marks]
- b) A study was carried out to estimate the mean height in centimetres of 15 years old student in Tunku Abdul Rahman secondary school. A random sample of 100 students was selected. Previous studies indicate that the variance of the height of such student is  $80\text{cm}^2$ . Suppose the sample mean height was  $\overline{X} = 145\text{cm}$ . Find a 95% confidence interval for the mean height of all 15 years old students in the school.

[7 marks]

# Question 2 (25 Marks)

a) There is a 1% chance that eggs packed in a box are cracked. A retailer will only purchase the eggs if there are not more than one cracked eggs in a sample of 30 eggs. What is the probability that the retailer will not make the purchase?

[7 marks]

Continued...

SMZ

- b) A company produces bags of a chemical, and it is concerned about impurity content. It was believed that the weights of impurities per bag are normally distributed with mean 12.2 grams and standard deviation 2.8 grams. A bag is chosen at random.
  - i. What is the probability that it contains less than 10 grams of impurities?

    [4 marks]
  - ii. What is the probability that it contains between 12 and 15 grams of impurities? [4 marks]
- c) The mean lifetime of 30 bulbs produced by Company A is 500 hours and the mean lifetime of 35 bulbs produced by Company B is 492 hours. If the standard deviation of all bulbs produced by Company A is 10 hours and the standard deviation of all bulbs produced by Company B is 15 hours, test at 1% significance level that the mean lifetime of bulbs produced by Company A is better than Company B.

[10 marks]

# Question 3 (30 Marks)

a) The marketing manager of a large supermarket chain would like to use shelf space (in feet) to predict the sales of pet food (in RM). A random sample of 12 equal sized stores is selected, with the following results:

#### **SUMMARY OUTPUT**

Regression Statistics							
Multiple R	-0.047057						
R Square	0.002214						
Adjusted R Square	-0.09756						
Standard Error	86.66026						
Observations	12						

#### ANOVA

	df	SS	MS	F	Significance F
Regression	1	166.6667	166.6667	0.022193	0.884537212
Residual	10	75100	7510		
Total	11	75266.67			

	Coefficients	Standard Error	t Stat	P-value	
Intercept	261.6667	61.27805	4.270153	0.001637	
Shelf space	-0.66667	4.475116	-0.14897	0.884537	

Continued...

- i. What are the dependent variable (y) and the independent variable (x) in the regression model above? [2 marks]
- ii. State the estimated linear regression equation.

[3 marks]

iii. State and interpret the slope coefficient, b<sub>1</sub>.

[3 marks]

iv. What is the correlation coefficient for the regression model? Interpret the value.

[3 marks]

- v. What is the coefficient of determination for the regression model? Interpret the value. [3 marks]
- vi. At 5% significance level, does the independent variable provide a significant contribution to the model? Discuss the result. [3 marks]
- vii. Predict the weekly sales of pet food for stores with 12 feet of shelf space for pet food.

  [3 marks]
- b) Incomes of a news agent in 2004 and 2005 were provided as following:

Type of News	200	)4	2005		
	Quantity (piece)	Income (RM)	Quantity (piece)	Income (RM)	
Daily paper	1763	1974.20	1540	2463.20	
Weekly paper	945	846.20	1180	1472.20	
Periodicals	441	747.60	860	1098.40	

i. Compute a weighted price index for the news agent income; Paasche Index and Laspeyres Index with 2004 as the base year. Explain the meaning of indices.

[8 marks]

ii. Using part (i), compute the Fisher's Ideal price index.

[2 marks]

End of Page

#### DESCRIPTIVE STATISTICS

Mean 
$$(\bar{x}) = \frac{\sum_{i=1}^{n} X_i}{n}$$

Standard Deviation (s) = 
$$\sqrt{\frac{\sum_{i=1}^{n} X_i^2}{n-1} - \frac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$$

Coefficient of Variation (CV) = 
$$\frac{\sigma}{\overline{X}} \times 100$$

Pearson's Coefficient of Skewness 
$$(S_k) = \frac{3(\overline{X} - Median)}{s}$$

#### **PROBABILITY**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$
 if A and B are independent

$$P(A \mid B) = P(A \text{ and } B) \div P(B)$$

## Poisson Probability Distribution

If X follows a Poisson Distribution, 
$$P(\lambda)$$
 where  $P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$ 

then the mean =  $E(X) = \lambda$  and variance =  $VAR(X) = \lambda$ 

## **Binomial Probability Distribution**

If X follows a Binomial Distribution B(n, p) where  $P(X = x) = {}^{n}C_{\nu}p^{\nu}q^{n-x}$ 

then the mean = E(X) = np and variance = VAR(X) = npq where q = 1 - p

#### **Normal Distribution**

If X follows a Normal distribution,  $N(\mu, \sigma)$  where  $E(X) = \mu$  and  $VAR(X) = \sigma^2$ 

then 
$$Z = \frac{X - \mu}{\sigma}$$

#### C. **EXPECTATION AND VARIANCE OPERATORS**

$$E(X) = \sum [X \circ P(X)]$$

$$VAR(X) = E(X^2) - [E(X)]^2$$
 where  $E(X^2) = \sum [X^2 \circ P(X)]$ 

where 
$$E(X^2) = \sum [X^2 \circ P(X)]$$

If 
$$E(X) = \mu$$
 then  $E(cX) = c \mu$ ,  $E(X_1 + X_2) = E(X_1) + E(X_2)$ 

If 
$$VAR(X) = \sigma^2$$
 then  $VAR(cX) = c^2 \sigma^2$ ,

If 
$$VAR(X) = \sigma^2$$
 then  $VAR(cX) = c^2 \sigma^2$ ,  $VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2 COV(X_1, X_2)$ 

where  $COV(X_1, X_2) = E(X_1X_2) - [E(X_1) E(X_2)]$ 

#### D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

 $(100 - \alpha)$  % Confidence Interval for Population Mean ( $\sigma$  Known) =  $\mu = \overline{X} \pm Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$ 

 $(100 - \alpha)$ % Confidence Interval for Population Mean (σ Unknown) =  $\mu = \overline{X} \pm t_{\alpha/2, n-1} \left( \frac{5}{\sqrt{n}} \right)$ 

 $(100 - \alpha)\%$  Confidence Interval for Population Proportion  $= \hat{p} \pm Z_{\alpha/2} \sigma_{p^*}$  Where  $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ 

Sample Size Determination for Population Mean  $= n \ge \left[\frac{(Z_{\alpha/2})\sigma}{E}\right]^2$ 

Sample Size Determination for Population Proportion =  $n \ge \frac{(Z_{\alpha/2})^2 \hat{p}(1-\hat{p})}{E^2}$ 

Where E = Limit of Error in Estimation

#### E. HYPOTHESIS TESTING

C	One Sample Mean Test
Standard Deviation (σ) Known	Standard Deviation (σ) Not Known
$7-\frac{x-\mu}{}$	$t = \frac{x - \mu}{x}$
$\omega = \sigma/\sqrt{n}$	$\sqrt[n]{\sqrt{n}}$

One Sample Proportion Test

$$z = \frac{\hat{p} - p}{\sigma_p} \qquad \text{where } \sigma_p = \sqrt{\frac{p(1 - p)}{n}}$$

Two Sample Mean Test

Standard Deviation (o) Known

$$z = \frac{\overline{(x_1 - x_2)} - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

Standard Deviation (c) Not Known  $t = \frac{\overline{(x_1 - x_2)} - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where  $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$ 

Two Sample Proportion Test

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)(\frac{1}{n_1} + \frac{1}{n_2})}}$$

where 
$$p = \frac{X_1 + X_2}{n_1 + n_2}$$

where  $X_1$  and  $X_2$  are the number of successes from each population

## F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model:  $Y = \beta_0 + \beta_1 X_1 + \varepsilon$ 

Sample Model:

$$\hat{y} = b_0 + b_1 x_1 + e$$

**Correlation Coefficient** 

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left((\sum X)^2 / n\right)\right]\left[\sum Y^2 - \left((\sum Y)^2 / n\right)\right]}} = \frac{COV(X, Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	1	SSR	MSR = SSR/1
Error/Residual	n – 2	SSE	MSE = SSE/(n-2)
Total	n-1	SST	

Test Statistic for Significance of the Predictor Variable

$$t_i = \frac{b_i}{S_{b_i}}$$
 and the critical value =  $\pm t_{\alpha/2,(n-p-1)}$ 

Where p = number of predictor

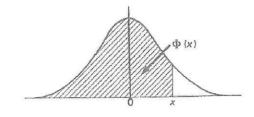
## G. INDEX NUMBERS

Simple Price Index	Laspeyres Quantity Index
$P = \frac{p_t}{p_0} \times 100$	$P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index	Paasche Quantity Index
Aggregate Price Index $P = \frac{\sum p_t}{\sum p_0} (100)$	$P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$
Laspeyres Price Index	Fisher's Ideal Price Index
$P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	√(Laspeyres Price Index)(Paa sche Price Index)
Paasche Price Index	Value Index
$P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	$V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$ .  $\Phi(x)$  is

the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When x < 0 use  $\Phi(x) = x - \Phi(-x)$ , as the normal distribution with zero mean and unit variance is symmetric about zero.



æ	$\Phi(x)$	x	$\Phi(x)$	æ	$\Phi(x)$	x	$\Phi(x)$	X	$\Phi(x)$	x	$\Phi(x)$
0.00	0.2000	0.40	0.6554	0.80	0.7881	1.20	0.8849	1.60	0.9452	2.00	0.97725
,ox	15040	·4I	-6591	·81	.7910	·2I	-8869	·61	.9463	·OI	.97778
.02	15080	.42	6628	.82	.7939	.22	-8888	·62	9474	.02	97831
.03	.5120	43	.6664	.83	.7967	.23	-8907	-63	.9484	.03	97882
.04	.2160	.44	.6700	-84	7995	.24	.8925	-64	.9495	.04	97932
0.07	0.5700	0.45	0.6736	0.85	0.8023	1.25	0.8044	z·65	0.9505	2.05	0.97982
0.02	0.2199 .239	.46	.6772	.86	.8051	-26	8962	.66	9515	.06	.98030
.07	5279	.47	-6808	.87	-8078	-27	-8980	-67	9525	.07	98077
.08	5319	.48	.6844	.88	-8106	-28	-8997	.68	9535	.08	.98124
.00	5359	.49	.6879	.89	.8133	29	.9015	.69	9545	.00	.98169
		7.7						WH - 500 Ph		0.50	0.08214
0.10	0.2308	0.20	0.6912	0.90	0.8159	1.30	0.0035	1.70	0.9554	2.10	98257
·II	.5438	.2x	.6950	-91	-8186	.31	.9049	'71	9564	II.	-98300
.13	5478	.52	-6985	-92	-8212	-32	.9066	'72	9573	.12	98341
.13	5517	. '53	.7019	.93	-8238	.33	.9082	'73	9582	13	98382
·14	*5557	54	.7054	.04	8264	*34	.9099	.74	.0591	14	90302
0.75	our rati	0.22	0.7088	0.02	0.8289	I:35	0.0115	1.75	0'9599	2.12	0.98422
0·15	.2636 .2636	.22	7123	-95	-8315	- 36	9131	.76	9608	.16	.98461
	.5675	.57	7157	.97	-8340	.37	9147	.77	.0616	.17	.98500
·17	5714	.58	7190	.98	-8365	.38	9162	.78	.9625	.18	.98537
.10	5753	'59	7224	.99	-8389	.39	.9177	.79	.9633	.10	-98574
				10000000	. 0	T. 45	0.0702	r·80	0.0641	2:20	0.08610
0.30	0.5793	0.60	0.7257	I.00	0.8413	I'40	0.0102	.8I	.9649	·2I	98645
·2I	.5832	·61	.7291	.OI	.8438	'4I	.9207	-82	9656	-22	-98679
.55	.5871	.62	.7324	.02	8461	'42	·9222 ·9236	-83	.9664	'23	98713
.23	.2910	.63	'7357	.03	8485	'43		-84	-9671	'24	98745
'24	.5948	·64	.7389	.04	·8508	'44	9251	049	9072	mong	20173
0.35	0.5987	0.65	0.7422	1.02	0.8531	1.45	0.9265	r.85	0.9678	2.25	0.98778
-26	6026	-66	.7454	.06	.8554	.46	.9279	-86	.9686	.26	198809
.27	.6064	.67	.7486	.07	.8577	.47	9292	.87	.9693	27	·98840
-28	-6103	.68	'7517	·08	8599	.48	.9306	-88	-9699	.28	-98870
.29	6141	.69	7549	.09	8621	.49	.0310	.89	.9706	'29	.98899
0.00	0.6170	0.70	0.7580	1.10	0.8643	1.20	0.9332	1.90	0.0713	2:30	0.08028
0.30	6217	.71	.7611	.II.	-8665	.2I	9345	.01	.9719	.31	98956
.31		1360000	•7642	.12	-8686	152	9357	-92	.9726	.32	98983
-32	.6255	.72	*7673	.13	-8708	'53	9370	.93	.9732	.33	.00010
.33	.6293	'73	(349 2036)	·14	.8729	.54	9372	-94	.9738	'34	-99036
.34	.6331	.74	'7704	77.00	0/29	54	9,500		,,,	٠.	
0.35	0.6368	0.75	9.7734	1.12	0.8749	1.55	0.9394	1.02		2.32	
.36	.6406	.76	100	.10	***	.56	.9406	.96	-9750	.36	
-37	6443	.77		17	8790	.57	.0418	.97		.37	
.38		.78		.18	-8810	-58	.9429	.98	200	.38	
.39	.6517	.79	200	.19	-8830	'59	9441	.99	.9767	.39	.99128
0.40	0.6554	0.80	0.7881	1.30	0.8849	1.60	0'9452	2.00	0.9772	2.40	0.99180

#### TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

DÇ	$\Phi(x)$	óc	$\Phi(x)$	ж	$\Phi(x)$	x	$\Phi(x)$	oc	$\Phi(x)$	oc	$\Phi(x)$
2.40	0.99180	2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99862	3.12	0.99918
'4I	.99202	.56	99477	-71	.99664	.86	.99788	·oz	-99869	16	199921
'42	.99224	.57	199492	.72	199674	.87	99795	'02	.99874	.17	99924
.43	99245	.58	199506	.73	199683	.88	.99801	.03	.99878	8r·	199926
.44	99266	·59	.99520	.74	•99693	.89	.99807	.04	199882	.19	.99929
2'45	0.99286	2.60	0.99534	2.75	0.99702	2.90	0.99813	3.02	0.99886	3.20	0.00031
.46	-99305	·61	'99547	.76	199711	.01	.99819	.06	.99889	'21	199934
47	99324	-62	.99560	.77	.99720	.92	.99825	07	.99893	.22	-99936
.48	'99343	-63	99573	.78	.99728	.93	·99831	.08	199896	.23	.99938
.49	.99361	-64	99585	.79	-99736	.94	199836	.09	.99900	.24	-99940
2.50	0.99379	2.65	0.99598	2.80	0.00744	2.95	0.99841	3.10	0.99903	3.25	0.99942
-51	.99396	.66	199609	·8r	99752	.96	.99846	II	.99906	.26	99944
52	.09413	.67	·99621	.82	-99760	.97	99851	12	.00010	127	99946
'53	99430	-68	-99632	.83	.99767	.98	199856	.13	99913	.28	-99948
54	99446	-69	-99643	.84	99774	.99	99861	.14	.99916	.29	.99950
2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.12	0.99918	3.30	0.99952

The critical table below gives on the left the range of values of x for which  $\Phi(x)$  takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of  $\Phi(x)$  indicated.

3.075 0.9990	3.263 0.9994	3.731 0.99990 3.750	3.916 0.99995 3.976 0.99996 4.055 0.99998
3.075 0.9990 3.105 0.9991 3.138 0.9992 3.174 0.9993 3.215 0.9994	3·320 0·9995 3·389 0·9996 3·480 0·9998 3·615 0·9999	3.759 0.99992 3.791 0.99993	4.055 0.99998
3.512 0.0003	3.612 0.0998	3.867 0.99994	4.173 0.99999 4.417 1.00000

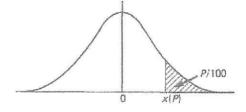
When x > 3.3 the formula  $1 - \Phi(x) = \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}} \left[ 1 - \frac{1}{x^2} + \frac{3}{x^4} - \frac{15}{x^6} + \frac{105}{x^8} \right]$  is very accurate, with relative error less than  $945/x^{10}$ .

# TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points  $\alpha(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2} dt.$$

If X is a variable, normally distributed with zero mean and unit variance, P/100 is the probability that  $X \ge x(P)$ . The lower P per cent points are given by symmetry as -x(P), and the probability that  $|X| \ge x(P)$  is 2P/100.



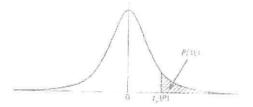
P	x(P)	P	x(P)	P	x(P)	P	x(P)	P	x(P)	P	x(P)
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0005
45	0'1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.0	2.3656	0.09	3.1214
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559
35	0.3823	4.4	1.2060	2.7	1.9268	1.7	2.1301	0.7	2:4573	0.07	3.1947
30	0.5244	4.3	1.7279	2.6	1.0431	1.6	2.1444	0.6	2.2131	0.00	3.5389
25	0.6745	4.0	1.7507	2.5	1.0600	1.5	2.1701	0.2	2.5758	0.02	3.2905
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.01	3.7190
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.002	3.8906
IO	1.2816	3.4	1.8250	2.2	2.0141	1.3	2.2571	0.3	2.8782	0.001	4.2649
5	1.6449	3.2	1.8522	2.1	2.0335	I.I	2.2904	O.I	3.0005	0.0002	4.4172

# TABLE 10. PERCENTAGE POINTS OF THE 4-DISTRIBUTION

This table gives percentage points  $t_p(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_p(P)}^{\infty} \frac{d\varepsilon}{(\varepsilon + \varepsilon^2/\nu)^{\frac{3}{2}(\nu+1)}}.$$

Let  $X_1$  and  $X_2$  be independent random variables having a normal distribution with zero mean and unit variance and a  $\chi^2$ -distribution with  $\nu$  degrees of freedom respectively; then  $t=X_1/\sqrt{X_2/\nu}$  has Student's t-distribution with  $\nu$  degrees of freedom, and the probability that  $t \geq t_{\nu}(P)$  is P/100. The lower percentage points are given by symmetry as  $-t_{\nu}(P)$ , and the probability that  $|t| \geq t_{\nu}(P)$  is 2P/100.



The limiting distribution of t as  $\nu$  tends to infinity is the normal distribution with zero mean and unit variance. When  $\nu$  is large interpolation in  $\nu$  should be harmonic.

P	40	30	25	20	15	EO	5	2'5	E	9.2	0.I	0.02
PEI	013249	0.7265	1,0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0007	1.386	1.886	8.050	4.303	6.062	9.025	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.185	4.241	5.841	10.31	12.02
4	0.2707	0.2686	0.7407	0.0410	1.100	1.233	2.132	2.776	3'747	4.604	7'173	8.610
		170 M 20 - 20		71	#15 <b>#</b> 15	500	64 m 20 m	- 7,0	3 /4/	of conf.	8 4 8 3	0.070
5	0.2672	0.5594	0.7267	0.0102	1.126	1.476	2.015	2.571	3.362	4.032	5.803	6.869
6	0.2648	0.2234	0.7176	0.9057	1.134	1.440	1.943	2'447	3'143	3.707	5'20'3	5.050
7	0.2632	0'5491	0.7111	0.8960	I.IIO	1'415	1.895	2.365		3'499	4.78	5'408
8	0.3610	0.2459	0.7064	0-8889	1.108	1'397	1.860	2.306	2.896	3'355	4.20	5041
9	0.5010	0'5435	0.7027	0.8834	1.100	1.383	1.833	2:262	2.821	3'250	4'29'	4.781
				0 50						- •		7 7
RO	0.2602	0.5415	0.6998	0-8791	1.003	1.372	K-8E2	2:228	2.764	3.169	4. E44	4:587
EE	0.2206	0.2399	0.6974	0.8755	1.088	1.363	1.796	2.301	2.718	3.100	4.02	4'437
12	0.5230	05386	0.6955	0.8726	1.083	1356	1782	2.179	2.681	3.055	3.930	4'318
13	0.3286	0.5375	0.6938	0.8702	1.070	1.320	1.771	2.190	2.650	3.012	3.85::	4.221
Id	0.5285	0.2366	0.6924	0.8681	1.026	1.342	1.761	2.142	2.624	2.977	3.78	4.140
				Novemb								
IS	0.2579	0.2324	0.6913	0.8662	1.074	1.341	1'753	5.131	2.602	2'947	3'73:	4.073
26	0.2576	0.2320	0.6601	0.8647	1.011	1.332	1.746	2.130	2.283	2.921	3.686	4.012
17	0.2573	0'5344	0.0892	0.8633	1.060	1.333	1.240	2.110	2.264	2.898	3.646	3.962
13	0.2571	0.2338	0.6884	0.8620	1.063	1.330	1.734	S.IOI	2:552	2.878	3.010	3.922
19	0.3269	0.2333	0.6876	0.8910	1.066	1.338	1.233	2.033	2.239	2.861	3.579	3.883
			22	22					12	20		22
20	0.3263	0.5329	0.6870	0.8600	1.064	1.352	I 725	2.086	2.528	2.845	3.223	3.820
21	0.5266	0.235	0.6864	0.8201	1.063	1.353	1.231	2.0go	2.218	2.831	3.23	3.810
22	0.2564	0.2351	0.6828	0.8283	1.001	1.331	1.212	2.074	5.208	5.810	3.202	3.792
23	0.2263	0.2314	0.6853	0.8575	1.000	1.310	1.414	2.000	2.200	2.80%	3.485	3.768
24	0.5265	0.2314	0.6848	0.8569	1.029	1.318	E-JIE	2.004	2.492	2.797	3.467	3.745
			<b></b>				0		•		160 1070 107466	
25	0.3291	0.2315	0.6844	0.8562	1.028	1.310	1708	2.060	2.485	2.787	3.450	3'725
26	0.5260	0.2300	0.6840	0.8557	1.028	1.312	1.706	2.056	2.479	2.779	3.435	3.404
27	0.2559	0.2300	0.6837	0.8551	1.022	1'314	1 703	2.022	2.473	2.771	3.421	3,600
28	0.2528	0.2304	0.6834	0.8546	1.026	1,313	E-70E	2,048	2.467	2:763	3.408	3.674
29	0.2557	0.2303	0.6830	0.8542	1.022	1.311	11699	2.042	2.462	2.756	3,396	3.620
			<b></b>	00	and the second second		al Cata				a. a.D.u	
30	0.2556	0.2300	0.6828	0.8338	1.055	1.310	1 697	2.042	2.457	2,750	3.382	3.646
32	0'2555	0.2297	0.6855	0.8530	1.024	1.300	1.694	2.037	2.449	2.738	3.362	3.622
34	0.5223	0.2294	0.6818	0.8523	1.023	1,304	1 691	2.032	2.441	2.728	3.348	3.QOI
36	0.5252	0.2291	0.6814	0.8217	1.023	1.300	1.688	5.058	2'434	2.410	3.333	3.282
38	0.5221	0.2288	0.6810	0.8212	1.021	1.304	1-686	2.024	2.429	2.413	3.310	3.266
						4.000	1 684	0.007	51166	A-RIA-I	010020	4.400
40	0.2550	0.2286	0.6807	0.8507	1.020	1.303	100	2.031	2.423	2.704	3.307	3.222
50	0'2547	0.2278	0.6794	0.8489	1.047	1.299	1 676	2,000	2.403	2:678	3.501	3.496
60	0.2545	0.2272	0.6786	0.8477	1.045	1,500	1.671	2.000	2.390		3'232	3.460
120	0.5239	0.2228	0.6765	0.8446	1.041	1.589	1-658	1.080	2.358	2.617	3.100	3'373
හ	0'2533	0.5244	0.6745	0.8416	1.036	1.383	1-645	1.060	2:326	2.576	3.000	3.501